CDS 230
Modeling and Simulation I

Module 9

Modeling Unequal Chances

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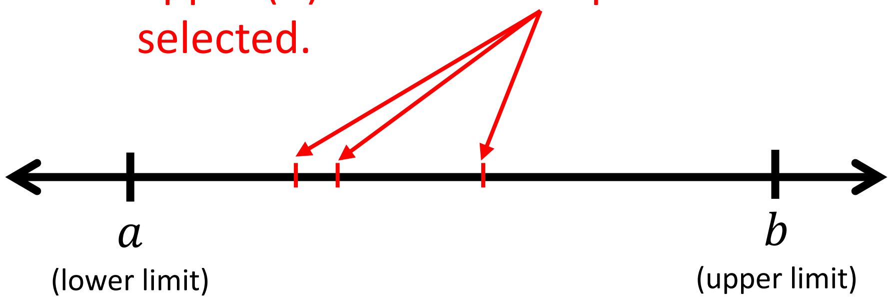
Recall modeling equal chances

Decimal numbers

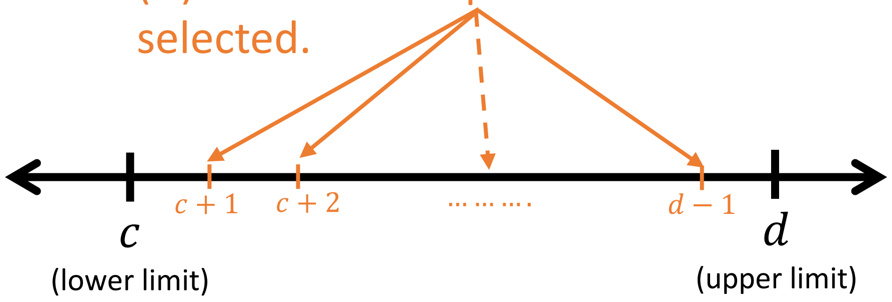
Any arbitrary points within lower (a) and upper (b) limits have equal chances to be selected.



`np.random.uniform(a,b)`

Integers

Any integers within lower (c) and upper (d) limits have equal chances to be selected.

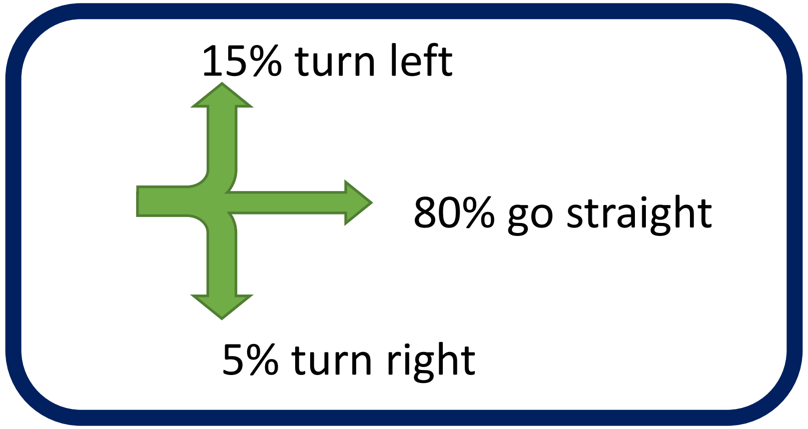


`np.random.randint(c,d+1)`

- Is equal chances sufficient to model all sorts of uncertainty?

How to model unequal chances?

- Recall the traffic flow example




```
chance = np.random.uniform(0, 100) # between 0 and 100

if chance <= 15.0: # first 15 percent => turn left
    print("Turn left")
elif 15.0 < chance <= 95.0:  # between 15 and 95 (80+15) percent => go straight
    print("Go straight")
else:
    print("Turn right") # the rest 5% (between 95 and 100) => turn right
```

How to model unequal chances easier?

- We can use `np.random.choice()` function which takes a list or NumPy array.

```
np.random.choice([1,2,3,4,5,6])
```

- The result is one of these six values, each with equal chances.
- Going back to the traffic flow example.

Each value represents one direction: left, straight, right

Each value represents the probability of occurrence of each direction respectively: left, straight, right

```
direction = [1,2,3]
probabilities = [0.15, 0.8, 0.05]
np.random.choice(direction, p=probabilities)
```

Don’t forget the argument name p.

Another approach to model unequal chances

Decimal numbers

- Identify quantities (or percentages) for different ranges.
  - 1 random number between 0 and 10.
  - 19 random number between 10 and 20.
  - 79 random number between 20 and 30.
  - 166 random number between 30 and 40.
  - 251 random number between 40 and 50.
  - 234 random number between 50 and 60.
  - 180 random number between 60 and 70.
  - …

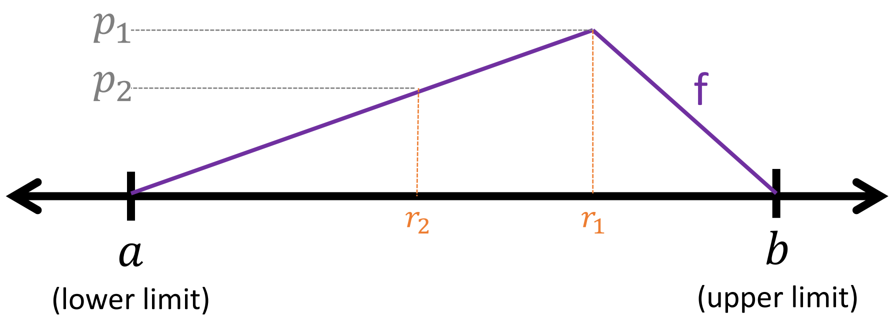
Integers

- Identify quantities (or percentages) for different integer values.
  - 30% chance for 1.
  - 20% chance for 2.
  - 10% chance for 3.
  - 5% chance for 4.
  - 3% chance for 5.
  - 3% chance for 10.
  - 1% chance for 11.
  - ….

Do you see a problem here?

A neat solution to model unequal chances

- Use probability distributions other than uniform.
- Probability distribution: A function that maps a value to a probability of occurrence.

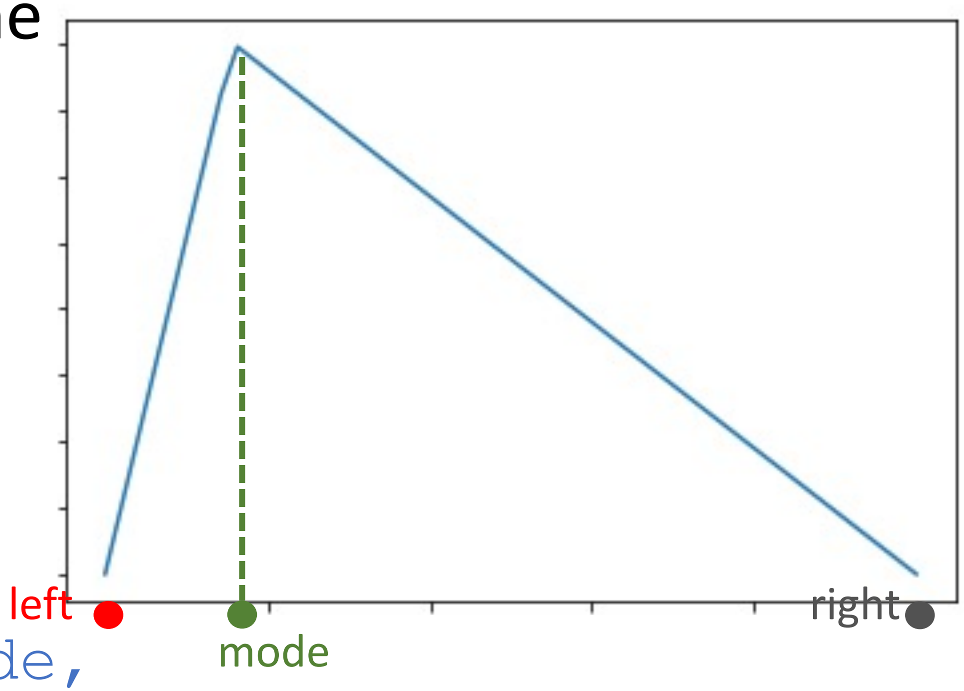


Here r_1 has a higher chance to be picked than r_2. Because…?

- We will go over three such non-uniform probability distributions
  - Triangular distribution
  - Normal distribution
  - Poisson distribution

Triangular distribution

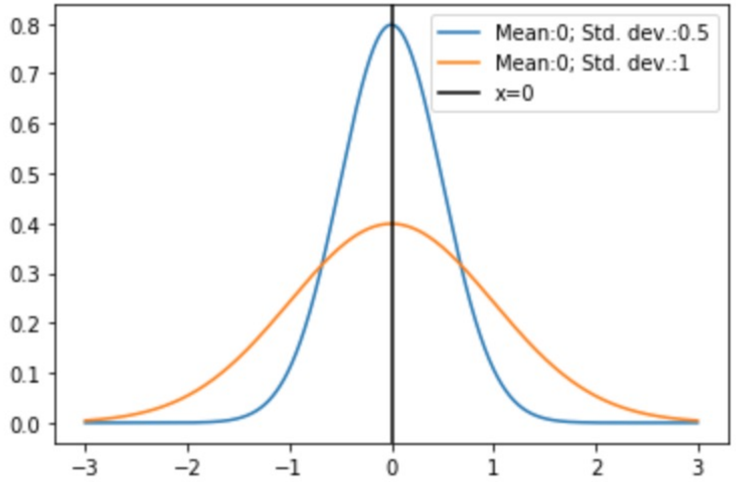
- Is used to model the probability of decimal numbers.
- Is used when we have a lack of data but some subjective knowledge about likely *minimum*, *maximum*, *likely* values.
- Needs three parameters:
  - *left*
  - *mode*
  - right
- NumPy syntax
  - `numpy.random.triangular(left, mode, right, size)`



Lets do live coding

Normal distribution

- Is used to model the probability of decimal numbers.
- Is "to go" distribution for unknown probabilities in many sciences.
- Needs two parameters:
  - *mean*
  - *standard deviation*.
- NumPy syntax
  - `numpy.random.normal(loc, scale, size)`



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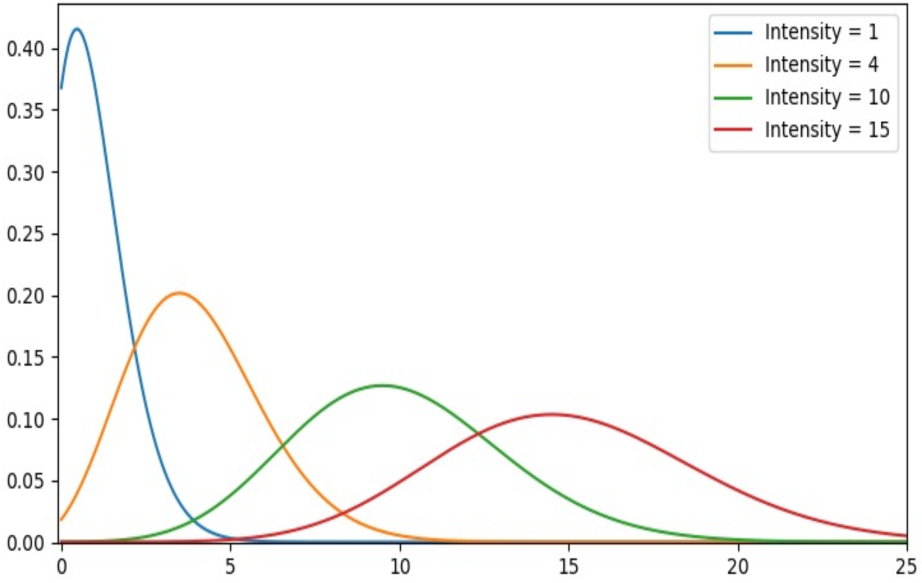
Poisson distribution

- Is used to model the probability of integer numbers.
- Is ”to go” distribution when modeling independent quantities.
  - Number of emails coming in an hour.
  - Number of people entering this building every hour.
- Needs only one parameter: *intensity* which is expected mean.
- NumPy syntax
  - `numpy.random.poisson(lam, size)`



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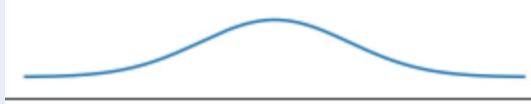
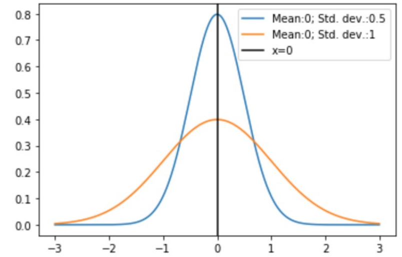
Terms you might see elsewhere

Term	Explanation
Probability	Chance
Probability distribution Statistical distribution	The shape of a probability space (how values are distributed)
Sampling	Drawing a value/a set of values from a probability distribution
Continuous numbers	Decimals
Discrete numbers	Integers
Continuous distribution	A distribution representing decimal numbers
Discrete distribution	A distribution representing integer numbers
Gaussian distribution Laplace-Gauss distribution Bell curve	Normal distribution





Other distributions you can sample from

- beta
- binomial
- chisquare
- dirichlet
- exponential
- f
- gamma
- geometric
- gumbel
- hypergeometric
- laplace
- logistic
- lognormal
- …

More comprehensive index: https://docs.scipy.org/doc/numpy-1.14.0/reference/routines.random.html

```
s = np.random.lognormal(3.0, 1.0, size=1000)
```

```
plt.hist(s,50)
plt.show()
```